Pergamon

0021-8928(95)00013-5

J. Appl. Maths Mechs, Vol. 59, No. 1, pp. 111–128, 1995

0021-8928/95 $24.00+0.00

A MODEL OF THE HYDRODYNAMIC INTERACTION BETWEEN INHOMOGENEITIES IN A FLUIDIZED BED†

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(*Received* 1 *June* 1994)

The stationary planar problem of the perturbed motion of the phases of a fluidized bed, which is generated by a rising or sinking formation consisting of an "adhering" circular bubble and a circular cluster of particles, is considered. An agglomerate of this type can serve as a model of the interaction between local inhomogeneities of a bed with different mean densities of the dispersed phase and, also, as a model of the transfer of solid particles in the bubble wake in a fluidized bed. The velocity and pressure fields of the fluid and solid phases over the whole flow domain are constructed. Conditions for the occurrence of circulatory flow patterns of the fluid phase outside the agglomerate and inside the bubble are investigated. Estimates of the velocity of motion of the agglomerate in the bed are obtained using the Davies–Taylor method.

It is known from experiment that the interaction of inhomogeneities in the distribution of a solid phase that differ in the concentration of particles can significantly distort the picture of the relative motion of the gas (liquid) and the particles in inhomogeneous fluidized systems. For example, the sinking motion of bubbles which are dragged to the base of the bed by having become stuck at the bottom to a dense cluster, that is, a so-called "beard" of particles, has been observed [1]. In an industrial column with a fluidized bed, the transport of particles by large bubbles can lead to macrocirculations of the dispersed phase and strong dynamic actions on the internal elements of the construction of the devices [2–4].

In their turn, the above hydrodynamic effects generate perturbations in the concentration and thermal fields, the analysis of which is an important element of the problem of raising the efficiency of the processes carried out in a fluidized bed.

There are a number of papers ([5–11] and others) which consider the problem of the hydrodynamic interactions of bubbles in a fluidized bed. Both pairwise interactions [8–11] as well as more complex systems of bubbles, that is, horizontal or vertical chains of them [5–7], are considered, and the analysis is confined to constructing the flow and pressure fields of the phases outside the bubbles.

In this paper a model of the pairwise interaction of inhomogeneities is extended to the case when there is a non-zero concentration of particles in one of them.

1. FORMULATION OF THE PROBLEM. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

Within the framework of the mechanics of continuous media a fluidized bed is identified with a double continuum, and the fluidizing agent and the solid particles are identified with interpenetrating and interacting continuous media, that is, the phases of the bed which are simulated as ideal fluids. Here, at a microlevel, the viscous interaction of the gas and the particles is described in the averaged equations of motion of the phases as a mass force of interphase friction. We shall confine ourselves to the case when the density of the fluidizing agent d_f is small compared with the density of the dispersed particles d_s: $d_f/d_s \ll 1$.

The shape of the agglomerate is shown in Fig. 1, where a_b and a_c are the radii of the bubble and of the cluster of particles, respectively. The agglomerate moves at a steady-state velocity U (in the laboratory system of coordinates) in a homogeneous, unbounded fluidized bed in a gravitational field with an acceleration g. The principal inertial system of coordinates S_1: $\{O, r, \varphi\}$ has its origin at the point where the bubble and the cluster are "stuck together". In addition to this system of coordinates, systems S_2 and S_3 are used in the analysis of the flow pattern inside the agglomerate. The latter two systems are obtained by a parallel translation of S_1 to the centre of the cluster and the bubble, respectively.

We shall assume that the distribution of the dispersed phase inside the cluster is spatially homogeneous and we shall confine ourselves to the case when there is no relative motion of the particles inside the cluster.

†*Prikl. Mat. Mekh.* Vol. 59, No. 1, pp. 121–139, 1995.

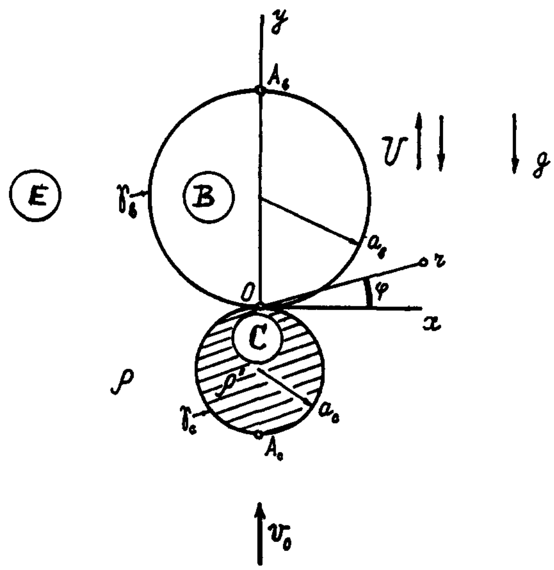

Fig. 1.

We now subdivide the whole flow domain into three subdomains: the exterior of the agglomerate (E), the interior of a bubble (B) and the interior of a cluster (packet) of particles (C). We shall write the equations of motion and continuity for the phases of the fluidized bed in the above-mentioned regions in the approximation of an inertia-less fluid phase ($d_f = 0$) and assuming that the interphase friction force is linear with respect to the velocity of relative motion of the phases:

$$\mathbf{v}-\mathbf{w} = -k(\varepsilon)\nabla p_f, \quad d_s\rho(\mathbf{w}\nabla)\mathbf{w} = -\nabla(p_f+p_s)+d_s\rho\mathbf{g} \tag{1.1}$$

$$\nabla\mathbf{v} = 0, \quad \nabla\mathbf{w} = 0$$

$$\nabla p_f' = 0 \rightarrow p_f' = p_{fb}'(t), \quad \nabla\mathbf{v}' = 0 \tag{1.2}$$

$$\mathbf{v}''-\mathbf{w}'' = -k''(\varepsilon'')\nabla p_f''$$

$$\nabla(p_f''+p_s'') = d_s\rho''\mathbf{g} \quad (\mathbf{w}'' \equiv 0), \quad \nabla\mathbf{v}'' = 0 \tag{1.3}$$

Here, $\mathbf{v}$, $\mathbf{w}$; p_f, p_s; ε, ρ are the velocities, pressures and volume concentrations of the fluid and solid phases, respectively, $\varepsilon + \rho = 1$; $k(\varepsilon)$ is the coefficient of permeability of the bed, $p_{fb}'(t)$ is the pressure of the fluid phase within a bubble which is constant in domain B at each instant of time t, and the flow parameters within a bubble (cluster) are denoted by a prime (two primes).

It follows from the second equation of (1.1) that the motion of the solid phase outside the agglomerate is the flow of an ideal fluid with a pressure $p_\Sigma = p_f + p_s$. This flow is assumed to be potential.

We denote the boundaries of the bubble and the cluster by γ_b and γ_c and formulate the boundary conditions which the fields $\mathbf{v}$, $\mathbf{w}$, p_f, p_s must satisfy

$$\gamma_b: \ \varepsilon v_n = v_n', \quad w_n = 0, \quad p_f = p_{fb}'(t), \quad p_s = 0 \tag{1.4}$$

$$\gamma_c: \ \varepsilon v_n = \varepsilon'' v_n'', \quad w_n = 0, \quad p_f + p_s = p_f'' + p_s'' \tag{1.5}$$

where the velocity components of the phases normal to the corresponding boundaries (the outward normal) are denoted by the subscript n.

As is usually done to these conditions we add the condition of the quasihomogeneity of the bed far from the agglomerate

$$\left.\frac{\partial p_f}{\partial y}\right|_\infty = -J = -\frac{\upsilon_0}{k(\varepsilon)} = -d_1\rho g \tag{1.6}$$

where $y = r\sin\varphi$ is the vertical Cartesian coordinate and υ_0 is the fluidization velocity, $g = |\mathbf{g}|$, and, also, the requirement of the boundedness of the phase velocities over the whole flow domain and the homogeneity of phase flows at an infinite distance from the agglomerate

$$\mathbf{w}|_\infty = \pm U\mathbf{i}_g, \quad \mathbf{v}|_\infty = (\pm U - \upsilon_0)\mathbf{i}_g, \quad \mathbf{i}_g = \frac{\mathbf{g}}{g} \tag{1.7}$$

Here and subsequently unless otherwise stipulated, the lower (upper) sign corresponds to the rising (sinking) agglomerate.

We now reduce the governing equations, the boundary conditions and the conditions at infinity to dimensionless form by taking the following scales: an arbitrary parameter l, which represents the size of the agglomerate, is taken as the length scale, the quantity Jl is taken as the pressure scale for the fluid phase, the quantity $p_{s\infty}$, the constant pressure in the solid phase in the quasihomogeneous bed remote from the agglomerate, is taken as the pressure scale for the solid phase, and the velocity U of its steady-state motion in the bed is taken as the velocity scale.

The dimensionless equations (1.1)–(1.3) have the form

$$\mathbf{v} - \mathbf{w} = -\delta^{-1}\nabla p_f, \quad \mathrm{Fr}(\mathbf{w}\nabla)\mathbf{w} = -\nabla(p_f + \sigma p_s) + \mathbf{i}_g \tag{1.8}$$

$$\nabla\mathbf{v} = 0, \quad \nabla\mathbf{w} = 0$$

$$\nabla p_f' = 0 \to p_f' = 0, \quad \nabla\mathbf{v}' = 0 \tag{1.9}$$

$$\mathbf{v}'' = -\frac{k''(\varepsilon'')}{k(\varepsilon)}\delta^{-1}\nabla p_f'', \quad \nabla(p_f'' + \sigma p_s'') = \frac{\rho''}{\rho}\mathbf{i}_g \tag{1.10}$$

$$\nabla\mathbf{v}'' = 0$$

$$\left(\delta = \frac{U}{\upsilon_0}, \quad \sigma = \frac{p_{s\infty}}{Jl}, \quad \mathrm{Fr} = \frac{U^2}{gl}\right)$$

The pressure of the fluid phase is measured from the quantity $p_{fb}'(t)$ so that the third boundary condition of (1.4) is replaced by the condition $p_f = 0$.

Next, the third boundary condition of (1.5) and the conditions at infinity (1.6) and (1.7) acquire the dimensionless form $p_f + \sigma p_s = p_f'' + \sigma p_s''$ and, correspondingly

$$\partial p_f / \partial y|_\infty = -1 \tag{1.11}$$

$$\mathbf{w}|_\infty = \pm\mathbf{i}_g, \quad \mathbf{v}|_\infty = (\pm 1 - \delta^{-1})\mathbf{i}_g \tag{1.12}$$

while the remaining conditions of (1.4) and (1.5) retain their previous form.

2. THE FLOW FIELD OF THE SOLID PHASE AND THE PRESSURE DISTRIBUTION OF THE FLUIDIZING FLUID

We will construct the potential velocity field $\mathbf{w}$ in the domain E outside the agglomerate. The complex potential for the flow around a planar contour of the shape under consideration by an ideal fluid will be sought by the method of conformal mapping onto a unit circle [12].

The required mapping, which preserves the orientation of the coordinate axes, is

$$z_1(z) = -iz_{10}\frac{z_{10}z_{11}(z) - z_{10}^{-1}}{z_{10}z_{11}(z) - z_{10}}, \quad z_1(\infty) = \infty \tag{2.1}$$

$$z_{10} = \exp\frac{ia}{a_b}, \quad z_{11}(z) = \exp\frac{2a}{z}, \quad a = \frac{\pi a_b a_c}{a_b + a_c}, \quad z = re^{i\varphi}$$

As an analogue of the complex flow potential in the z_1-plane, let us consider the characteristic function for the flow of an ideal fluid, which is homogeneous at infinity, with a unit velocity in the negative (positive) direction of the vertical axis, around a unit circle. We have

$$W(z_1) = \pm i\left(z_1 - \frac{1}{z_1}\right) \tag{2.2}$$

The complex potential $W_s(z)$ for the flow around the boundary $\gamma_b \cup \gamma_c$ of inhomogeneity differs from the function $W(z_1(z))$ solely by a factor which ensures that the first condition at infinity of (1.12) is satisfied. On combining relationships (2.1) and (2.2) and taking account of what has been said, we obtain

$$W_s(z) = \pm a\left(\sin\frac{a}{a_b}\right)^{-1}\left[\zeta(z) + \frac{1}{\zeta(z)}\right], \quad \zeta(z) = \operatorname{sh}\left(\frac{a}{z} + i\frac{a}{a_b}\right)\left(\operatorname{sh}\frac{a}{z}\right)^{-1} \tag{2.3}$$

On separating out the imaginary part in (2.3), we find the stream function of the solid phase in domain E

$$\operatorname{Im} W_s(z) = \psi_s(r,\varphi) = \pm\frac{a \operatorname{sh} u}{\operatorname{ch} u - \cos\upsilon}\,\frac{\cos\upsilon - \cos(2a/a_b - \upsilon)}{\operatorname{ch} u - \cos(2a/a_b - \upsilon)} \tag{2.4}$$

$$u = \frac{2a\cos\varphi}{r}, \quad \upsilon = \frac{2a\sin\varphi}{r}$$

In the limiting case when $a_c \to 0$ or $a_b \to 0$, expressions (2.3) and (2.4) describe the complex potential and the stream function of the flow around single circles of radii a_b and a_c with centres at the points ia_b, $-ia_c$, respectively.

The pressure distribution of the fluid phase in domain E is constructed by a method which is analogous to the method previously used in [13]. As follows from Eqs (1.8)–(1.10), the function p_f is harmonic over the whole flow domain. By virtue of the uniqueness of the solution of the external Dirichlet problem $\Delta p_f = 0$, $p_f|_{\gamma_b} = 0$, the pressure distribution of the fluid phase outside the agglomerate is precisely the same as in the problem of the motion in the bed of a single bubble of radius a_b.

Let us now consider the analytic function $\Phi(z) = p_f^* + ip_f$, where p_f^* is a function which is harmonically conjugate with p_f. We identify $\Phi(z)$ with the complex potential of the flow, in a direction which is antiparallel to the real axis, of a stream of ideal fluid around the contour of the bubble (on this contour, which is a "streamline", $\operatorname{Im}\Phi(z) = p_f = 0$ in accordance with the third boundary condition of (1.4) on γ_b) since, as follows from condition (1.4), the relation

$$d\Phi/dz|_\infty = \partial p_f/\partial y|_\infty = -1$$

must be satisfied far from the agglomerate.

On mapping the cavity of the bubble onto the unit circle in the same was as above, we obtain

$$\Phi(z) = -\left(z - ia_b + \frac{a_b^2}{z - ia_b}\right) \tag{2.5}$$

whence it follows that the required pressure distribution of the fluid phase in domain E has the form

$$p_f(r,\varphi) = -(r\sin\varphi - a_b)\left(1 - \frac{a_b^2}{r^2 - 2a_b r\sin\varphi + a_b^2}\right) \tag{2.6}$$

The function which is harmonically conjugate with p_f is defined by the expression

$$p_f^*(r,\varphi) = \operatorname{Re}\Phi(z) = -r\cos\varphi\left(1 + \frac{a_b^2}{r^2 - 2a_b r\sin\varphi + a_b^2}\right) \tag{2.7}$$

Note that, among the boundary conditions (1.5), there is no condition for the balancing of the normal stresses in the fluid phase on the boundary of the cluster which, in the inertia-less approximation, reduces to the continuity of the pressure p_f on γ_c. This condition has been considered previously [14, 15] when analysing the motion of single inhomogeneities (clusters of particles with different concentrations) in a fluidized bed. While being non-contradictory in simple cases, it is now incompatible with the assumptions regarding the properties and motions of the phases which have led to the system of equations of motion and continuity (1.8)–(1.10) in the more complex model of the interaction of inhomogeneities which is being considered here.

Actually, the velocity field of the fluid phase in domain E is uniquely constructed (the first equation of (1.8)) by invoking relationships (2.4) and (2.6) which yield the velocity field of the solid phase and the pressure of the fluid outside the agglomerate. Under the assumption that $p_f = p_f''$ on γ_c, we obtain that the fluid phase pressure field inside the cluster is also described by expression (2.6). Then, the fluid velocity in domain C is also uniquely found from the first equation of (1.10) and it is impossible to satisfy the natural condition of the conservation of the flow rate of the fluid phase through the discontinuity γ_c (the first boundary condition of (1.5)).

The fact is that the system of boundary conditions (1.4) and (1.5) with the inclusion of the condition of p_f on γ_c mentioned above, which is written in the approximation $d_f = 0$, does not take account of the inertial effects on solid phase concentration discontinuities. Meanwhile, we know [16, 17] that similar effects have a significant influence on the flow patterns even in rarefied systems.

In order to take account of the above-mentioned effects indirectly, let us assume that a transition layer (the thickness of which is negligibly small compared with the size of the agglomerate) is formed on the boundary of the cluster γ_c and that the inertial terms in the equations of motion of the fluid phase predominate in this transition layer. In this case, the solution in the transition layer, which can be obtained using the methods proposed in [18], matches the pressure distributions p_f and p_f'' so that, without taking account of the transition layer, the field p_f is discontinuous on the boundary of the cluster.

The harmonic function p_f'', which is now not associated with the condition of continuity on γ_c, is found by solving a Neumann problem with the boundary condition

$$\gamma_c:\quad \frac{\partial p_f''}{\partial n} = -\frac{k\delta}{k''}\upsilon_n'' = -\frac{k\varepsilon\delta}{k''\varepsilon''}\upsilon_n = \frac{\varepsilon k}{\varepsilon'' k''}\frac{\partial p_f}{\partial n}$$

since, on the boundary of the cluster $\upsilon_n = -\delta^{-}(\delta p_f/\delta n)$ and, by virtue of its impermeability in the case of the dispersed phase: $w_n|_{\gamma_c} = 0$. Here, the criterion for the Neumann problem to be solvable [19], which has the form

$$\oint_{\gamma_c} \upsilon_n'' dl = \oint_{\gamma_a} \upsilon_n dl = 0$$

is satisfied since it has the simple physical meaning that there are no sinks or sources of the fluid phase in domain E.

It is convenient to obtain the distribution p_f'' initially in the system S_2, since the equation for the boundary of the cluster has the simplest form in this system: γ_c: $r = a_c$, so that $\partial/\partial n = \partial/\partial r$. By displacing the origin of coordinates to the centre of the cluster and using expression (2.6), we obtain

$$\left.\frac{\partial p_f}{\partial n}\right|_{\gamma_c} = -\chi(\varphi),\quad \chi(\varphi) = \sin\varphi + \lambda^2\frac{[(\lambda+1)^2+1]\sin\varphi - 2(\lambda+1)}{[(\lambda+1)^2 - 2(\lambda+1)\sin\varphi + 1]^2},\quad \lambda = \frac{a_b}{a_c} \tag{2.8}$$

The harmonic function p_f'' can now be established, apart from a constant, using Dini's integral [20]

$$p_f''(r,\varphi) = -\frac{a_c}{\pi}\int_{-\pi}^{\pi}\left.\frac{\partial p_f''}{\partial n}\right|_{\gamma_c}\ln|z_1 - z|\,dt =$$

$$= \frac{a_c}{2\pi}\frac{\varepsilon k}{\varepsilon'' k''}\int_{-\pi}^{\pi}\chi(t)\ln[r^2 + a_c^2 - 2ra_c\cos(\varphi - t)]dt \tag{2.9}$$

($z_1 = a_c e^{it}$ and the previous notation is retained for the new variables).

Returning to the initial frame of reference, we can write the pressure distribution of the fluid phase inside the cluster in the form

$$p_f''(r,\varphi)=\frac{a_c}{2\pi}\frac{\varepsilon k}{\varepsilon''k''}\int_{-\pi}^{\pi}\chi(t)\ln[r^2+2ra_c\sin\varphi+2a_c^2-2ra_c\cos(\varphi-t)-2a_c^2\sin t]dt \qquad (2.10)$$

3. THE FLUID PHASE FLOW FIELD

We will construct the fields **v**, **v**′, **v**″ using the results obtained in Section 2 and the governing equations (1.8)–(1.10). In fact, they determine the pattern of the perturbed gas flow in the neighbourhood of the agglomerate and the local characteristics of heat- and mass-transfer processes in the fluid phase.

The flow of the fluidizing agent outside the agglomerate. It follows from the first equation and the last two equations of (1.8) that the stream functions of the phases are connected in domain E by the relationship $\Psi_f = \Psi_s + \delta^{-1}p_f^*$. Hence, from expressions (2.4) and (2.5), we obtain

$$\Psi_f(r,\varphi)=\pm\frac{a\,\mathrm{sh}\,u}{\mathrm{ch}\,u-\cos\upsilon}\frac{\cos\upsilon-\cos(2a/a_b-\upsilon)}{\mathrm{ch}\,u-\cos(2a/a_b-\upsilon)}-\delta^{-1}r\cos\varphi\left(1+\frac{a_b^2}{r^2-2ra_b\sin\varphi+a_b^2}\right) \qquad (3.1)$$

An analysis of this relationship shows that, as in the simpler models [13, 14], a so-called "cloud" may be formed in the neighbourhood of the interacting bubble and cluster, which is a region of closed circulation of the fluidizing agent. The boundary of this region is impermeable to the fluid phase and represents a zone of enhanced resistance to mass transfer into the bed. The clouds formed around fairly rapidly rising agglomerates, that is, when the inequality $\upsilon_0 < U < +\infty$ $(1 < \delta < +\infty)$ holds. If the velocity of ascent of an inhomogeneity is less than the fluidization velocity: $U < \upsilon_0$, or, when it is sinking, no domain of closed circulation encompassing the agglomerate is formed.

The boundary of the cloud is defined by the equation

$$\frac{a\,\mathrm{sh}\,u}{\mathrm{ch}\,u-\cos\upsilon}\frac{\cos\upsilon-\cos(2a/a_b-\upsilon)}{\mathrm{ch}\,u-\cos(2a/a_b-\upsilon)}-\delta^{-1}r\cos\varphi\left(1+\frac{a_b^2}{r^2-2ra_b\sin\varphi+a_b^2}\right)=0 \qquad (3.2)$$

When $U = \upsilon_0$ $(\delta = 1)$, the cloud occupies the whole of the domain E and, at higher rising velocities its boundary is located close to the surface $\gamma_b \cup \gamma_c$ of the agglomerate, with which it coincides when $U \gg \upsilon_0$ $(\delta \to +\infty)$.

In the other limiting case $a_c \to 0$, it follows from formula (3.2) (after changing to the system of coordinates S_3) that

$$r(\delta)=a_b\left(\frac{\delta+1}{\delta-1}\right)^{1/2}$$

which is the well-known result obtained by Davidson [21] for the radius of the cloud around a single bubble of radius a_b.

The change in the configuration of the cloud as a function of the velocity of ascent of the agglomerate is shown in Fig. 2 for the case when $a_b = a_c = 1$ (the values $\delta^{-1} = 0.1, 0.4, 0.6, 0.7, 0.8$ correspond to curves 1–5).

The streamlines of the fluid phase in domain E, which are described by the stream function (3.1), are shown in Figs 3–5 for a cluster radius $a_c = 1$ and a bubble radius $a_b = 2$ (Fig. 3), $a_b = 1$ (Fig. 4) and $a_b = 1/3$ (Fig. 5) for three sets of flow conditions: fast rising $(\delta^{-1} = 0.4)$, slow rising $(\delta^{-1} = 1.1)$ and sinking $(\delta^{-1} = 0.5)$.

The values of the stream function, in order of increasing curve number in Figs 3–5, are: $\psi_f = 0.8$, 0.4, 0 (the cloud boundary), –0.3, –0.5, –0.53, –0.57, –0.8, –1.2, –1.4, –1.6 (Fig. 3), $\psi_f = 0, -0.3, -0.5$, –0.7, –0.9, –1.03, –1.1, –1.28, –1.3, –1.33, –1.5, –1.7, –1.9, –2.1, –2.2 (Fig. 4), $\psi_f = 0, -0.15, -0.3, -0.45$, –0.6, –0.9, –1.5 (Fig. 5).

It can be seen that closures of the flow of the fluid phase on the surface of the bubble and the cluster are a characteristic feature of the rising regimes. However, in the case of fast rising $(0 < \delta^{-1} < 1)$, perturbations of the form shown are localized within the boundary of the domain of closed circulation. Outside this domain the geometry of the streamlines corresponds to the flow around a cloud as a kernel

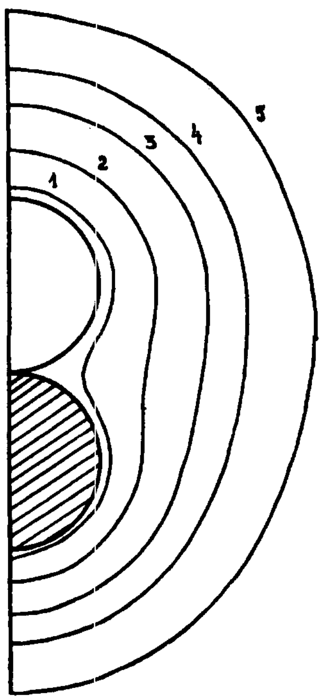

Fig. 2.

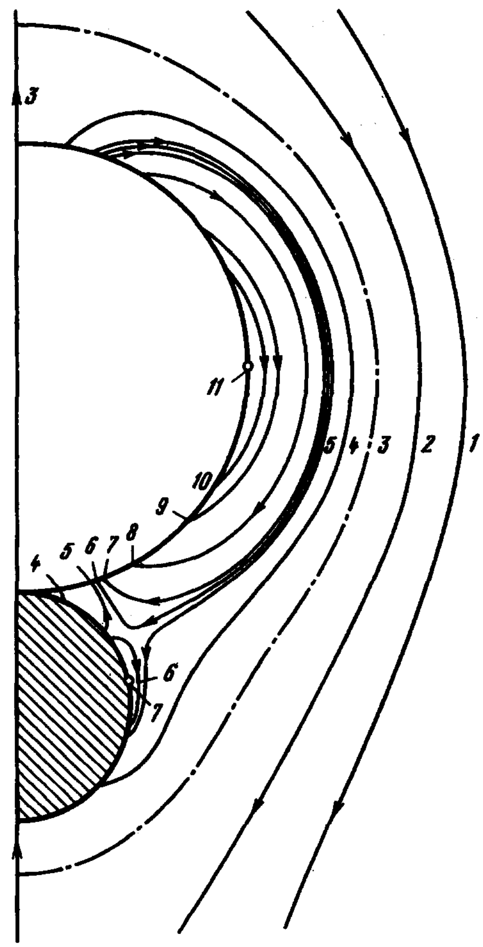

Fig. 3.

with a solid boundary which is given by (3.2). In the limiting case $U \gg v_0$, when the boundaries of the cloud and the agglomerate are the same, the closures which have been described have no effect on the transport of the reagents from an inhomogeneity to adjacent segments of the bed. Slowly rising agglomerates perturb the uniform flow of the fluid phase more significantly, and large regions of the bed adjoining its boundary are involved in convective gas exchange with the inhomogeneity.

Note that, as in the case of interacting bubbles [8], there is a fluid flow which binds the interacting bubble and cluster. It is partly concentrated in the neighbourhood of the point where their boundaries join and also (in the case of rising agglomerate) in the neighbourhoods of the upper and lower extremities of the agglomerate.

Sinking inhomogeneities, as is obvious from the results presented in Fig. 5, only slightly perturb the uniform gas flow which maintains the bed in a fluidized state.

The flow of the fluid phase inside the bubble. The second equation of (1.9) is the principal equation which describes the gas flow inside the bubble. This equation defines the required flow field non-uniquely. This is due to losses of information concerning the vorticity of the flow in domain B in the inertia-less approximation $d_f = 0$. Here, in order to take account directly of the inertial nature of the fluid phase, we specify that rot $\mathbf{v}' = \mathbf{a}$, within the bubble, where $\mathbf{a}$ is an arbitrary solenoidal vector. After this, as we know [22], it is possible to establish the vector $\mathbf{v}'$ itself uniquely using its normal component $v'_n|_{\gamma_b} = \varepsilon v_n|_{\gamma_b}$, which is found from the first boundary condition (1.4) using the result (3.1). The supplementary condition for this problem to be solvable $\oint_{\gamma_b} v'_n dl = \iint_B \nabla v' ds$ is satisfied in view of the solenoidal nature of the field $\mathbf{v}'$ and the fact that there are no singularities inside the bubble.

The simplest solenoidal vectors, which are considered below as $\mathbf{a}$, are $\mathbf{a=0}$ and $\mathbf{a}$ = **const**. The first case corresponds to irrotational flow of the fluid phase inside the bubble and the second to its flow

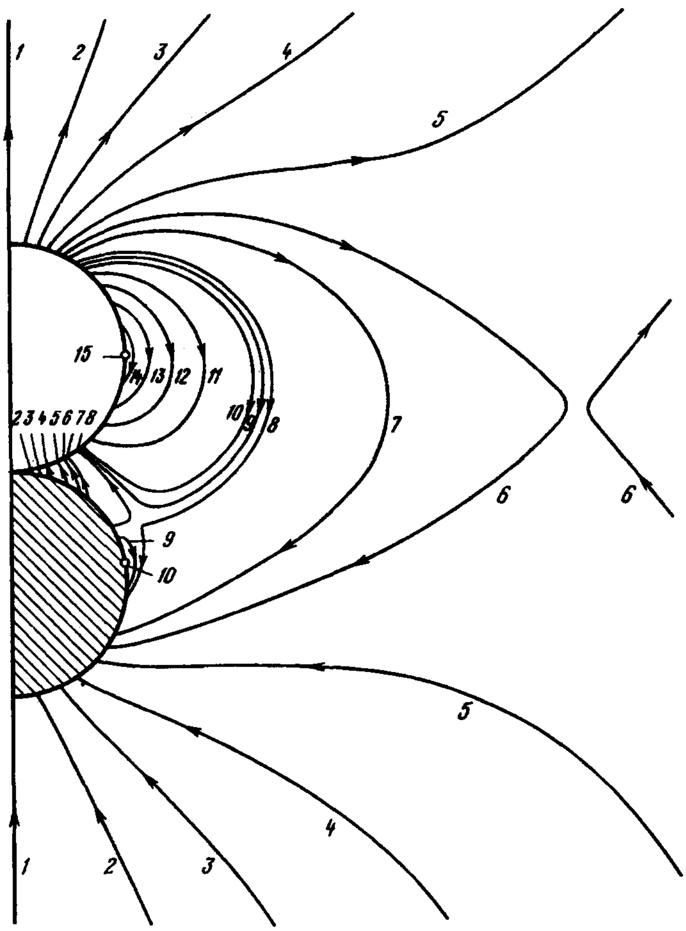

Fig. 4.

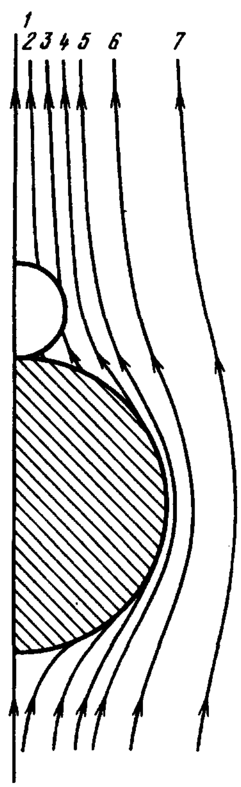

Fig. 5.

with a constant vorticity [23] (in both cases, the fluid phase velocity $\mathbf{v}'$ is finite everywhere inside the bubble).

1. $\mathbf{a} = 0$. In this case $\mathbf{v}' = \nabla\varphi_f'$, where φ_f' is the potential of the gas flow in the domain B. Since the fields $\mathbf{v}, \mathbf{w}, p_f$ outside the agglomerate are related by the first equation of (1.8) and, on the bubble surface, the field $\mathbf{w}$ satisfies the boundary condition $w_n = 0$, we obtain the following Neumann problem for finding the potential φ_f'

$$\Delta\varphi_f' = 0, \quad z \in B; \quad \gamma_b: \ \frac{\partial \varphi_f'}{\partial n} = -\varepsilon\delta^{-1}\frac{\partial p_f}{\partial n} \tag{3.3}$$

The solution of problem (3.3), which is found using the Dini integral by means of relationships (2.5) and (2.6) in the frame of reference S_3 with the origin at the centre of the bubble, has the simple form

$$\varphi_f'(r,\varphi) = 2\varepsilon\delta^{-1} r \sin\varphi \tag{3.4}$$

As a result of (3.4), for the stream function of the fluid phase inside the bubble, we obtain

$$\psi_f'(r,\varphi) = \varphi_f'^{*}(r,\varphi) = -2\varepsilon\delta^{-1} r\cos\varphi = -2\varepsilon x \tag{3.5}$$

where $x = r\cos\varphi$ is a Cartesian coordinate in S_3.

Hence, irrotational flow of the fluid phase inside the bubble is a homogeneous flow with a velocity $2\varepsilon\upsilon_0$ which runs through the bubble in an upwards direction from below in all forms of the agglomerate motion (see Fig. 6).

As might have been expected, we note that, since the boundary condition for the conservation of the flow rate of the fluid phase is satisfied on the discontinuity γ_b, the resulting flow is topologically consistent with the gas flow outside the agglomerate (in this case the stream function ψ_f is discontinuous on the boundary of the bubble). Actually, it is clear from (3.1) that the streamlines of the external flow, which intersect with γ_b "pierce" the bubble at points lying on vertical straight lines.

2. We will now consider a more complex model of the flow of the fluid phase inside the bubble and, in fact, we will assume that curl $\mathbf{v}' = \mathbf{a} \neq \mathbf{0}$, $\mathbf{a} = -\mathbf{k}\Gamma$, where $\mathbf{k}$ is the unit vector normal to the plane of flow and Γ is a parameter which characterizes the vortex strength in domain B. It now follows from the basic equation $\nabla \mathbf{v}' = 0$ that the stream function of the fluid phase inside the bubble satisfies Poisson's equation

$$\Delta\psi'_f = \Gamma \tag{3.6}$$

Remaining in the frame of reference S_3, we introduce the auxiliary function

$$\Psi(r,\varphi) = \psi'_f(r,\varphi) - \Gamma r^2 / 4$$

into the treatment.

Then, as a result of the fact that $\Delta_{r,\varphi} r^2 = 4$, from Eq. (3.6) we obtain

$$\Delta\Psi = 0 \tag{3.7}$$

that is, Ψ is harmonic function in domain B. Next, writing the condition for the conservation of the flow rate of the fluid phase on the boundary of the bubble in terms of the stream function ψ'_f, we arrive at the following boundary condition for the function Ψ:

$$\gamma_b: \quad \Psi = -2a_b\varepsilon\delta^{-1}\cos\varphi - \Gamma a_b^2 / 4 \tag{3.8}$$

The solution of problem (3.7), (3.8) with the appropriate condition at infinity has the form

$$\Psi(r,\varphi) = -2\varepsilon\delta^{-1} r\cos\varphi - \Gamma a_b^2 / 4$$

whence we obtain the following relation for the stream function

$$\psi'_f(r,\varphi) = \Gamma(r^2 - a_b^2)/4 - 2\varepsilon\delta^{-1} r\cos\varphi \tag{3.9}$$

Since, in the model of constant vorticity of the field $\mathbf{v}'$ inside the bubble being considered, there is a preferred direction which is determined by the sign of Γ, the streamlines cease to be symmetrical about

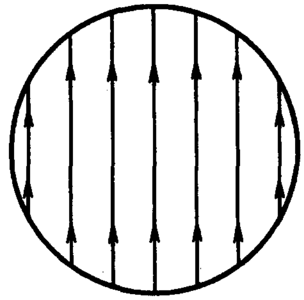

Fig. 6.

the ordinate axis (there is symmetry about the abscissa axis). They are arcs of concentric circles with centre at the point $(4\varepsilon\delta^{-1}/\Gamma, 0)$ on the abscissa axis.

The gas flow pattern inside the bubble in the case when $\Gamma > 0$ is shown in Fig. 7 (when $\Gamma < 0$, it is a specular reflection in the ordinate axis) for $a_b = 1$. If $4\varepsilon\delta^{-1}/\Gamma \geqslant a_b = 1$ (a small positive vorticity Γ), the streamlines are weakly curved arcs (Figs 7a and b). In the limit as $\Gamma \to 0$, we obtain streamline pattern of irrotational flow (Fig. 6). If, however, the vortex scale exceeds the boundary value $\Gamma_* = 4\varepsilon\delta^{-1}$, there is a circular zone with closed streamlines inside the bubble, and the zone size is greater for greater Γ (Fig. 7c). In the limit as $\Gamma \to +\infty$, the circulation domain fills the whole bubble from within. Its radius is given by the expression $a_{cl} = 1 - 4\varepsilon\delta^{-1}/\Gamma$.

If the parameter Γ is fixed, then, for agglomerates with velocities of ascent $U > 4\varepsilon\upsilon_0/\Gamma$, the flow of the fluid phase in domain B is non-circulating while, when $U < 4\varepsilon\upsilon_0/\Gamma$, it is a circulatory flow.

3. In conclusion, we will consider a discontinuous vorticity distribution of the fluid phase in domain B of the following form: $\operatorname{rot} \upsilon' = -\operatorname{sign}(x)\mathbf{k}\Gamma$ and $\Gamma > 0$ as in case 2. The auxiliary function Ψ is now introduced by the equality $\Psi = \psi'_f \pm \Gamma r^2/4$, since $\Delta\psi'_f = \pm\Gamma$ (alternation of the signs corresponds to a transition from the right half of the bubble to the left half). The boundary conditions for the function Ψ on the bubble surface take the form

$$\gamma_b \; : \; \Psi = -2a_b\varepsilon\delta^{-1}\cos\varphi \mp \Gamma a_b^2/4 \tag{3.10}$$

The solution of the boundary-value problem (3.7), (3.10) with the appropriate condition at infinity yields the following expression for the stream function of the fluid phase

$$\psi'_f(r,\varphi) = \begin{cases} -2\varepsilon\delta^{-1}r\cos\varphi - \dfrac{\Gamma a_b^2}{\pi}\displaystyle\sum_{n=0}^{\infty}\left(\frac{r}{a_b}\right)^{2n+1}\frac{(-1)^n}{2n+1}\cos(2n+1)\varphi \pm \frac{\Gamma}{4}r^2, & x \neq 0 \\ 0, \quad x = 0 \end{cases} \tag{3.11}$$

and, moreover, here the convergence of the series can be speeded up using well-known methods [20].

It follows from (3.11) that the pattern of the streamlines of the fluidizing agent in the case of the vorticity distribution inside the bubble under consideration is symmetrical about the Cartesian coordinate axes. The nature of the dependence of the flow pattern on the magnitude of the parameter Γ is analogous to that considered above in case 2. The critical value Γ_* of the vorticity which subdivides the non-circulatory $(0 < \Gamma \leqslant \Gamma_*)$ and circulatory $(\Gamma_* < \Gamma < +\infty)$ patterns of the gas flow through the bubble is as follows: $\Gamma_* = 4\pi\varepsilon\delta^{-1}/(\pi - 1)$.

The streamlines of the fluid phase in the bubble under these two sets of conditions are shown in Figs 8a and b for a value of $\varepsilon\delta^{-1} = 0.45$. Here, $\Gamma_* \cong 2.64$ and the magnitudes of the vorticity scale in the cases indicated are taken as being equal to $\Gamma = 0.5$ and 10, respectively, and $a_b = 1$. The values of the stream function on the curves in increasing order are: $\psi'_f = 0, -0.2, -0.4, -0.6, -0.8, -0.9$ (Fig. 8a) and $\psi'_f = 0, -0.1, -0.4, -0.6, -0.9, -1.05, -1.2, -1.3, -1.35, -1.36$ (Fig. 8b).

The boundary of the circulation domain is defined by (3.11) when $\psi'_f = \mp 2\varepsilon\delta^{-1}a_b$. In the limit as $\Gamma \to \infty$, symmetric vortices fill the corresponding halves of the bubble. It becomes impermeable to the external flow, and the additional resistance to the transfer of reagents in the fluid phase is concentrated on its boundary. The limiting position of the streamline, which is generated at a point on the abscissa axis, is specified by the equation $r^2 + r - 2/\pi = 0$, which has a single positive root $r_{**} \cong 0.51$.

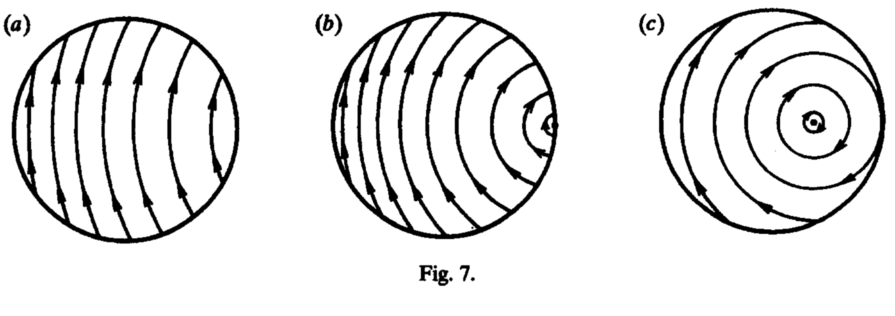

Fig. 7.

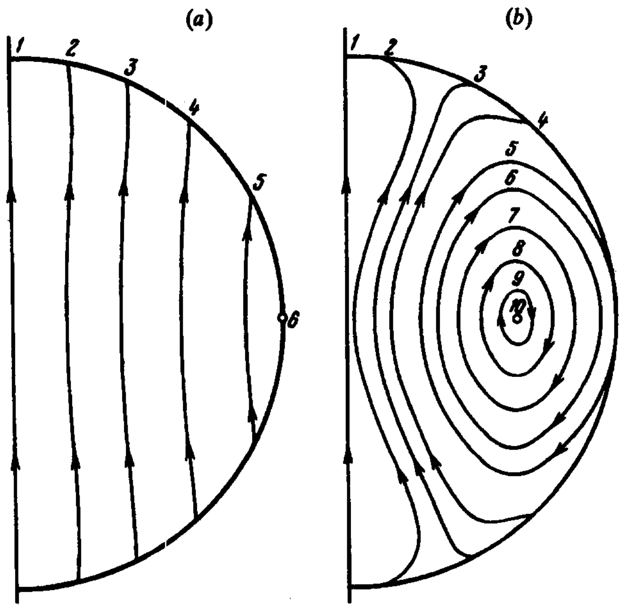

Fig. 8.

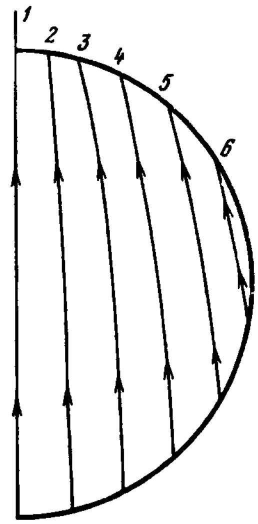

Fig. 9.

More complex models of the vorticity distribution inside a bubble such as, for example, the competition between vortices Γ_1 and Γ_2 when they have a different strength and direction, can also be treated in the same way as in cases 1–3. The values of the free parameters in models of this type can be determined from some additional conditions on the agglomerate boundary (certain continuity conditions [23], for example) which relate the external and internal flows of the fluid phase.

Gas flow inside the cluster. It follows from the first equation of (1.1) that the relation between the stream function and the pressure of the fluid phase in domain C is given by $\psi'_f = -k''k^{-1}\delta^{-1}p_f''^*$. Using (2.9), we find that, in system S_2

$$\psi'_f(r,\varphi) = \frac{a_c}{\pi}\,\frac{\varepsilon}{\varepsilon''}\,\delta^{-1} I(r,\varphi) \tag{3.12}$$

$$I(r,\varphi) = -\int_{-\pi}^{\pi} \chi(t)\arg(z_1 - z)\,dt$$

The level lines of the integral $I(r, \varphi)$ are shown in Fig. 9 and provide a complete representation regarding the potential flow of the fluidizing fluid inside the cluster of particles. The corresponding values of the functions $I(r, \varphi)$ on curves 1–6 are 0, –0.78, –1.56, –2.32, –3.06 and –3.59.

The irrotational gas flow in domain C, where the solid particles are fixed with respect to one another, is characterized by the absence of closed streamlines under all forms of motion of the agglomerate. More complex (and, in the case of low concentrations ρ'', more realistic) models of the particles inside a cluster and, in particular, a model of the solid phase motion inside a cluster with a constant vorticity similar to that considered for a bubble above may be treated. In the models, the question of the possibility of the closure of the streamlines of the fluidizing fluid within a cluster requires further investigation.

4. THE PRESSURE DISTRIBUTION FOR THE SOLID PHASE. ESTIMATE OF THE AGGLOMERATE VELOCITY IN THE BED

Starting from the analogy between a fluidized bed and conventional fluids [1–3], let us assume that the direction of motion of an agglomerate in a bed is defined by the ratio of the Archimedes force and the gravity force acting on the particles which form the cluster and which are concentrated in domain C. Let S_b and S_c be the areas of domains B and C, respectively. Then, an agglomerate rises (sinks) in the bed when the condition

$$(S_b + S_c)\rho \lesseqgtr \rho'' S_c \tag{4.1}$$

is satisfied or, what is the same, when $\rho''/\rho \gtrless 1 + \lambda^2$. In particular cases of the motion of a bubble or of a cluster, relation (4.1) remains true. The situation when the agglomerate is in equilibrium in the fluidized system corresponds to equality of the left- and right-hand sides of (4.1).

The equations of motion of the solid phase outside the agglomerate and inside the cluster have simple integrals

$$p_\Sigma = p_f + \sigma p_s = p_{\Sigma 0} + (\mathbf{i}_g, \mathbf{r}) - \frac{\mathrm{Fr}}{2}\mathbf{w}^2 \tag{4.2}$$

$$p_\Sigma = p_f'' + \sigma p_s'' = p_{\Sigma 0}'' + \frac{\rho''}{\rho}(\mathbf{i}_g, \mathbf{r}) \tag{4.3}$$

where $p_{\Sigma 0}$, $p_{\Sigma 0}''$ are certain constants.

In the model under consideration as well as in simpler problems concerning the motion of inhomogeneities in a fluidized bed (and even in single phase fluids) [13, 15], the condition for the balance of the normal stresses in the solid phase cannot be satisfied everywhere on the boundary $\gamma_b \cup \gamma_c$ by means of expressions (4.2) and (4.3). Here, we shall use the Davies–Taylor procedure [24] which is usual in similar cases, that is, we shall construct a solution which satisfies the above-mentioned conditions locally in the neighbourhood of leading critical points (points A_b and A_c for a rising and sinking agglomerate, respectively) as the result of a suitable choice of the Froude number which characterizes the rate of motion of the agglomerate in the system.

The velocity distributions of the solid phase on the surface of the bubble and the cluster are found from relation (2.4) and have the form (system S_1)

$$\begin{aligned} \mathbf{w}^2|_{\gamma_b} &= \left(\frac{a}{a_b}\right)^4 \frac{1}{\sin^4\varphi}\sin^2\frac{a}{a_b}\,\mathrm{sh}^2\left(\frac{a}{a_b}\mathrm{ctg}\,\varphi\right)\left[\mathrm{ch}\left(\frac{a}{a_b}\mathrm{ctg}\,\varphi\right) - \cos\frac{a}{a_b}\right]^{-4} \\ \mathbf{w}^2|_{\gamma_c} &= \left(\frac{a}{a_c}\right)^4 \frac{1}{\sin^4\varphi}\sin^2\frac{a}{a_b}\,\mathrm{sh}^2\left(\frac{a}{a_c}\mathrm{ctg}\,\varphi\right)\left[\mathrm{ch}\left(\frac{a}{a_c}\mathrm{ctg}\,\varphi\right) + \cos\frac{a}{a_b}\right]^{-4} \end{aligned} \tag{4.4}$$

Determining the constants in (4.2) and (4.3) from the continuity conditions $p_\Sigma|_{\gamma_b}(A_b) = 0$, $p_\Sigma|_{\gamma_c}(A_c) = p_\Sigma''|_{\gamma_c}(A_c)$ and subsequently expanding the jumps in the total pressure p_Σ on the boundaries of the bubble and the cluster close to the corresponding critical points using expressions (4.4), we arrive at the following estimates for the velocities of ascent (4.5) and descent (4.6) of the agglomerate

$$U = 2\sqrt{g a_b}\left(\frac{a_b}{a}\right)^3\left(1 - \cos\frac{a}{a_b}\right)^2\left(\sin\frac{a}{a_b}\right)^{-1} \tag{4.5}$$

$$U = 2\sqrt{g a_c}\left(\frac{a_c}{a}\right)^3\left(1 + \cos\frac{a}{a_b}\right)^2\left(\sin\frac{a}{a_b}\right)^{-1}\sqrt{\frac{\rho''}{\rho} - 1} \tag{4.6}$$

As $a_c \to 0$, Eq. (4.5) has the form $U = 1/2\sqrt{(g a_b)}$ and is identical with the well-known result obtained by Davidson for a single circular bubble [21]. In the other limiting case when $a_c \to 0$, the estimate of the velocity of descent of a single circular cluster with a solid phase concentration $\rho'' > \rho$: $U_c = 1/2\sqrt{(g a_c(\rho''/\rho - 1))}$ follows from (4.6). A cluster with the limiting density ($\rho'' = 1$) sinks with a terminal velocity $U_c(\rho'' = 1) = 1/2\sqrt{(g a_c(1 - \rho)/\rho)}$.

5. THE CASE OF A NON-DEGENERATE BOUNDARY BETWEEN THE BUBBLE AND THE CLUSTER

The formulation of the problem of an interacting bubble and cluster proposed in Section 1 can be extended to the case when they are deformed so that the shape of the agglomerate is a combination of two segments of circles of radii a_b and a_c which have a common base of length $2h$ (Fig. 10). As the system of coordinates, let us take a system of bipolar coordinates (ξ, η): $\xi = \theta_1 - \theta_2$, $\eta = \ln(r_2/r_1)$ with poles at the points $(\pm h, 0)$ on the abscissa axis of the Cartesian system xOy.

The degree of deformation of the bubble and the cluster is represented by the parameters $n_b \in [0, 1]$, $n_c \in [1, 2]$: $h = a_b \sin \pi n_b = -a_c \sin \pi n_c$.

The system of equations of motion and continuity of the phases in domains E, B and C as well as the conditions at infinity are retained in their previous form (1.8)–(1.12). To the system of boundary conditions, we add the conditions of conservation of flow rate of the fluid phase and of the balance of the normal stresses on the segment of the boundary γ between the bubble and the cluster

$$\gamma:\ \upsilon_n' = \varepsilon''\upsilon_n'',\quad p_f''=0,\quad p_f'=0 \tag{5.1}$$

The conformal mapping of the interior of the lune, which specifies the shape of the agglomerate, onto the unit circle, which is analogous to the mapping (2.1), has the form [25]

$$z_1(\zeta) = ie^{ib}\frac{e^{i\beta}-e^{-ib}}{e^{i\beta}-e^{ib}},\quad z_1(\infty)=\infty \tag{5.2}$$

where

$$b=\pi\frac{2-n_c}{2-n},\quad \beta=\frac{\zeta+\pi(2-n_c)}{2-n},\quad \zeta=\xi+i\eta$$
$$n=n_c-n_b,\quad z=ih\operatorname{ctg}\frac{\zeta}{2}$$

The limiting form of this expression as $h \to 0$ (a_b and a_c are fixed) is identical with (2.1).

Next, the analogue of the complex flow potential (2.3) of the solid phase in domain I outside the agglomerate is written, using (5.2), in the form

$$W_s(\zeta)=\mp\frac{h}{(2-n)\sin b}\left[\frac{\operatorname{sh}\alpha}{\operatorname{sh}(\alpha-ib)}+\frac{\operatorname{sh}(\alpha-ib)}{\operatorname{sh}\alpha}\right] \tag{5.3}$$
$$\alpha=i\left(\frac{\zeta}{4-2n}+b\right)$$

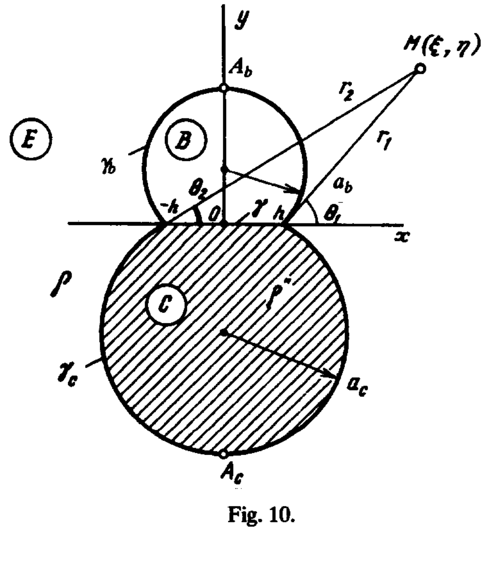

Fig. 10.

and the corresponding expression for the stream function is obtained by separating out the imaginary part in (5.3)

$$\Psi_s(\xi,\eta)=\pm\frac{h}{2-n}\,\frac{\operatorname{sh}u}{\operatorname{ch}u-\cos\upsilon}\,\frac{\cos\upsilon-\cos(2b+\upsilon)}{\operatorname{ch}u-\cos(2b+\upsilon)} \tag{5.4}$$

$$u=\frac{\eta}{2-n},\quad \upsilon=\frac{\xi}{2-n}$$

By mapping the segment of the bubble onto the unit circle, we can construct the analogue of the potential (2.5)

$$\Phi(\zeta)=\frac{ih}{(1+n_b)\sin\alpha_0}\left(\frac{\operatorname{sh}\alpha_1}{\operatorname{sh}\alpha_2}-\frac{\operatorname{sh}\alpha_2}{\operatorname{sh}\alpha_1}\right)$$

$$\alpha_0=\frac{\pi}{1+n_b},\quad \alpha_1=\frac{i\zeta}{2(1+n_b)},\quad \alpha_2=\frac{i(\zeta+2\pi)}{2(1+n_b)}$$

Hence the pressure distribution of the fluid phase outside the agglomerate and the corresponding adjoint function have the form

$$p_f(\xi,\eta)=-\frac{h}{1+n_b}\left(\operatorname{ctg}\frac{\pi}{1+n_b}+\frac{\sin\upsilon_1}{\operatorname{ch}u_1-\cos\upsilon_1}\right)\frac{\cos\upsilon_1-\cos(\alpha_0+\upsilon_1)}{\operatorname{ch}u_1-\cos(\alpha_0+\upsilon_1)} \tag{5.5}$$

$$u_1=\frac{\eta}{1+n_b},\quad \upsilon_1=\frac{\xi}{1+n_b}$$

$$p_f^*(\xi,\eta)=-\frac{h}{1+n_b}\,\frac{\operatorname{sh}u_1}{\operatorname{ch}u_1-\cos\upsilon_1}\,\frac{2\operatorname{ch}u_1-\cos\upsilon_1-\cos(\alpha_0+\upsilon_1)}{\operatorname{ch}u_1-\cos(\alpha_0+\upsilon_1)} \tag{5.6}$$

By combining relationships (5.4) and (5.6), it is possible to obtain an expression for the stream function of the fluid phase, analogous to (3.1) and which, in the limit as $h\to 0$, reduces to it. As before, a cloud with closed streamlines only exists in domain E when $0\leqslant\delta^{-1}<1$, that is, in the case of inhomogeneities which are rising sufficiently rapidly.

Below, we shall consider the special case $n_c = 1 + n_b$ ($n = 1$) when the agglomerate is a circle of radius $a_b = a_c$, the upper part of which is occupied by the bubble and the lower part is occupied by the cluster of particles. Such a model of a bubble with its wake has been investigated, for example, in [25] when analysing the rheological properties of fluidized systems. Putting $a_b = a_c = 1$, we find

$$\Psi_f(r,\varphi)=\pm r\cos\varphi\frac{r^2-2r\sin\varphi\cos\pi n_b-\sin^2\pi n_b}{r^2-2r\sin\varphi\cos\pi n_b+\cos^2\pi n_b}-$$

$$-\delta^{-1}\frac{\sin\pi n_b}{1+n_b}\operatorname{sh}u_1\left[\frac{1}{\operatorname{ch}u_1-\cos\upsilon_1}+\frac{1}{\operatorname{ch}u_1-\cos\left(\dfrac{2\pi}{1+n_b}+\upsilon_1\right)}\right] \tag{5.7}$$

where, in accordance with (5.5) and the definition of bipolar coordinates

$$\operatorname{th}[u_1(1+n_b)]=\operatorname{th}\eta=\frac{2rh\cos\varphi}{r^2+h^2}$$

$$\operatorname{tg}[\upsilon_1(1+n_b)]=\operatorname{tg}\xi=\frac{2rh\sin\varphi}{r^2-h^2}$$

where r and φ are polar coordinates (Fig. 10).

The flow pattern of the fluid phase outside the agglomerate in a circulatory regime when $\delta^{-1} = 0.9$ is shown in Fig. 11 as an example for $n_b = 0.5$; the corresponding values of ψ_f for curves 1–10 are 0, –0.1, –0.4, –0.7, –1.0, –1.2, –1.3, –1.39, 0.1 and 0.2.

Note that, in the model of an interacting bubble and cluster under consideration, the field **w** has no singularities on the boundary $\gamma_b \cup \gamma_c$ only if $n \in (1, 2]$, that is, in the case of "apple-like" shapes of agglomerate. On "lentil-like" contours ($n \in [0, 1)$), this field is unbounded in the neighbourhood of the poles of the bipolar system. As far as the velocity field **v** of the fluid phase is concerned, this always has singularities at the points of the boundary of the agglomerate indicated. The fact is that the segment of the bubble when all $n \in [0, 2]$ has two sharp edges (an internal angle $< \pi$). The singularity in the field **v** referred to here is a consequence of the general theory of mappings of contours with corner points [12, 27].

As previously, the internal boundary-value problems in domains B and C are correct. For instance, for a fluid phase pressure p_f'' inside the cluster, we obtain, basing on the conditions (1.5) and (5.1), the mixed problem

$$\Delta p_f'' = 0, \quad p_f''|_\gamma = 0, \quad \left.\frac{\partial p_f''}{\partial n}\right|_{\gamma_c} = \frac{k\varepsilon}{k''\varepsilon''}\left.\frac{\partial p_f}{\partial n}\right|_{\gamma_c}$$

which has a unique solution [28]. The velocity field v'' is now determined by the first equation of (1.10) and $\oint_{\gamma_c \cup \gamma} \mathrm{v}_n'' dl = 0$ since there are no fluid phase sinks and sources inside the cluster. On satisfying the conditions of conservation of the fluid phase flow rate on the boundaries γ_c and γ, we find from this (the integrals converge)

$$0 = \int_{\gamma_c} \mathrm{v}_n'' dl + \int_{\gamma} \mathrm{v}_n'' dl = \frac{\varepsilon}{\varepsilon''} \int_{\gamma_c} \mathrm{v}_n dl + \frac{1}{\varepsilon''} \int_{\gamma} \mathrm{v}_n' dl = \frac{1}{\varepsilon''}\left(\varepsilon \int_{\gamma_c} \mathrm{v}_n dl + \int_{\gamma} \mathrm{v}_n' dl\right) \qquad (5.8)$$

On now taking account of the fact that $\oint_{\gamma_b \cup \gamma_c} \mathrm{v}_n'' dl = 0$ for the external fluid phase flow field and, also, the fact that the outward normals to the closed contours $\gamma_b \cup \gamma$ and $\gamma_c \cup \gamma$ on the contour γ are opposite, we obtain

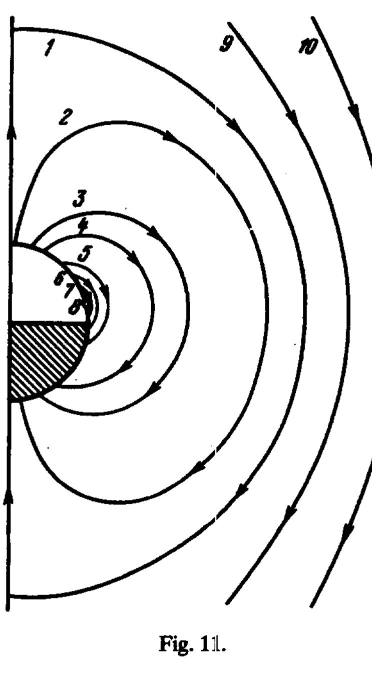

Fig. 11.

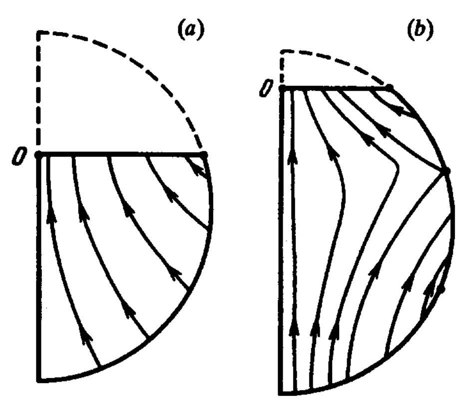

Fig. 12.

$$\oint_{\gamma_b \cup \gamma} v'_n dl = \int_{\gamma_b} v'_n dl + \int_{\gamma} v'_{-n} dl = \varepsilon \int_{\gamma_b} v_n dl - \int_{\gamma} v'_n dl = -(\varepsilon \int_{\gamma_c} v_n dl + \int_{\gamma} v'_n dl) = 0$$

and the field $\mathbf{v}'$ inside the bubble can be established using the earlier scheme (Section 3).

The results of a qualitative analysis of the irrotational field $\mathbf{v}''$ inside the cluster ($n = 1$) are shown in Fig. 12. As follows from relations (5.4) and (5.6), the streamlines of the fluid phase on the side of the cluster are normal to the boundary γ between it and the bubble. Furthermore, an investigation of the stream function (5.7) on the boundary of the agglomerate shows that, as the fraction of the bubble in the agglomerate becomes smaller, the flow pattern undergoes a qualitative change when $n_b = n_{b*} = [\pi - \arccos 3/4]/[\pi + \arccos 3/4]$. The gas flow through the cluster for $0 < n_b < n_{b*}$ (Fig. 12a) is replaced, when $n_{b*} < n_b < 1$, by a more complex flow: part of the flow departs from the cluster into the surrounding bed while another part enters into the cluster and, subsequently, into the bubble (Fig. 12b).

As previously, the solid phase pressure distribution in domain E is described by expressions (4.2) and (4.3). Here, by virtue of the second boundary condition of (5.1) and the fact that $(\mathbf{i}_g, \mathbf{r})|_\gamma = 0$, the boundary condition $p''_s = 0$ is exactly satisfied on the common segment of the boundary between the bubble and the cluster.

The velocity distribution of the dispersed phase on the boundary of the agglomerate has the form

$$\mathbf{w}^2|_{\gamma_b} = \frac{4(\operatorname{ch}\eta - \cos \pi n_b)^2}{(2-n)^4} \frac{\sin^2 b \operatorname{sh}^2 \frac{\eta}{2-n}}{\left(\operatorname{ch}\frac{\eta}{2-n} + \cos b\right)^4}$$

$$\mathbf{w}^2|_{\gamma_c} = \frac{4(\operatorname{ch}\eta - \cos \pi n_c)^2}{(2-n)^4} \frac{\sin^2 b \operatorname{sh}^2 \frac{\eta}{2-n}}{\left(\operatorname{ch}\frac{\eta}{2-n} - \cos b\right)^4}$$

On satisfying the third condition of (1.5) for p''_s in the neighbourhood of the critical points A_b and A_c, we obtain, by the Davies–Taylor method, the following estimates for the rising (5.9) and sinking (5.10) agglomerate (in dimensional form)

$$U = \frac{(2-n)^3(1+\cos b)^2(\cos \pi n_b + 1)}{2 \sin b \sin \pi n_b (1 - \cos \pi n_b)} \sqrt{g a_b} \tag{5.9}$$

$$U = -\frac{(2-n)^3(1+\cos b)^2(\cos \pi n_b + 1)}{2 \sin b \sin \pi n_c (1 - \cos \pi n_c)} \sqrt{g a_c \left(\frac{\rho''}{\rho} - 1\right)} \tag{5.10}$$

which reduce to (4.4) in the limit as $h \to 0$.

Condition (4.1) for a rising (sinking) agglomerate in the model under consideration has the form

$$\frac{\rho''}{\rho} \leqslant 1 - \lambda^2 \frac{\pi(1-n_b) + \sin \pi n_b \cos \pi n_b}{\pi(1-n_c) + \sin \pi n_c \cos \pi n_c} \tag{5.11}$$

The estimates (4.5), (4.6), (5.9) and (5.10) are illustrated in Fig. 13 for $\rho'' > \rho$ when, depending on whether conditions (4.1) and (5.11) are satisfied, the agglomerate can rise or sink in the bed. In the first case, the stability of the frontal part of the bubble is ensured at a rising velocity which is determined by means of (4.5) and (5.9) and only by the configuration of the inhomogeneity and is independent of the concentration of particles in the cluster. In the second case, the dependence of the velocity of steady sinking of the agglomerate on its density reduces to a factor $\sqrt{(\rho''/\rho - 1)}$ in formulae (4.6) and (5.10).

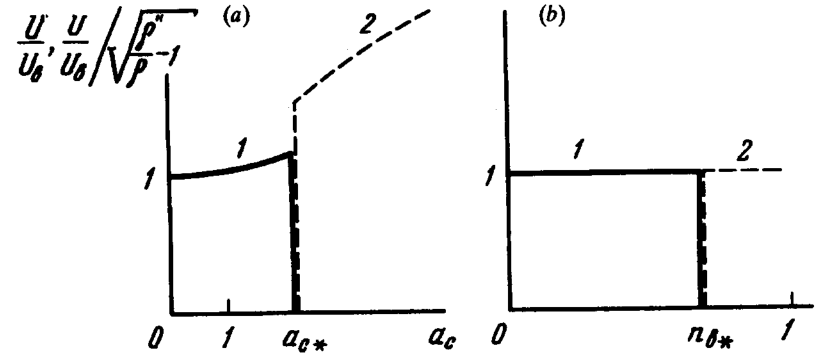

Fig. 13.

Curves of the ratios U/U_b (curve 1) and $U/(U_b\sqrt{(\rho''/\rho - 1)})$ (curve 2) of the velocities of the agglomerate and of a single bubble of radius a_b as a function of the size of cluster a_c for $\rho = 0.5$, $\rho'' = 0.6$, $a_b = 1$ are shown in Fig. 13a.

The boundary value a_{c*}, which separates rising inhomogeneities ($a_c < a_{c*}$) from sinking ones ($a_c > a_{c*}$) is defined by the equation $\lambda^2 + 1 = \rho''/\rho$. The threshold effect of the concentration of particles in a cluster on the velocity of motion of an agglomerate is a characteristic feature of the curves in Fig. 13a. The fact that there is no other relationship between its velocity of ascent and ρ'', apart from the balancing relation (4.1), is justified within the framework of the theory of ideal fluids, and the model which excludes internal motion of particles in the cluster when the solid phase slip on its surface is permitted and the external pressure field p_s depends solely on the geometrical characteristics of the inhomogeneity.

In the case of a non-degenerate boundary between the bubble and the cluster in the special case when $n = 1$, which was considered above, n_b is the sole shape factor determining the configuration of an inhomogeneity. In accordance with condition (5.1), its threshold value, which separates rising agglomerates from sinking ones, is given by the equation $2\pi n_b - \sin 2\pi n_b - 2\pi\rho''/\rho = 0$ which, when $\rho'' > \rho$, has a root $n_{b*} \in (0, 1)$ (for $\rho = 0.5$, $\rho'' = 0.6$, we have $n_{b*} \cong 0.69$). In this case, the velocity of ascent (descent) of an agglomerate according to estimates (5.9) and (5.10) is identical with the velocity of ascent of a bubble (the velocity of descent of a single cluster) of equal size (Fig. 13b, curves 1 and 2).

REFERENCES

1. TODES O. M. and TSITOVICH O. B., *Apparatus with a Fluidized Granular Bed*. Khimiya, Leningrad, 1982.
2. GEL'PERIN N. I., AINSHTEIN V. G. and KVASHA V. B., *Principles of Fluidization Engineering*. Khimiya, Moscow, 1967.
3. DAVIDSON J. F. and HARRISON D. (Eds), *Fluidization*. Academic Press, London, 1971.
4. KUNII D. and LEVENSPIEL O., *Fluidization Engineering*. Wiley, New York, 1969.
5. COLLINS R., An extension of Davidson's theory of bubbles in fluidized beds. *Chem. Eng. Sci.* **20**, 8, 747–766, 1965.
6. GABOR J. D., Interaction effects on the fluid dynamics of bubbles in a fluidized bed: chain of rising bubbles in an infinite two-dimensional medium. *Ind. Eng. Chem. Fundam.* **8**, 1, 84–91, 1969.
7. GABOR J. D. and KOPPEL L. B., Interaction effects on the fluid dynamics of bubbles in a fluidized bed: chain of rising bubbles in an infinite three-dimensional medium. *Chem. Eng. Progr. Symp. Ser.* **66**, 105, 28–36, 1970.
8. CLIFT R., GRACE J. R., CHEUNG L. and DO T. H., Gas and solids motion around deformed and interacting bubbles in fluidized beds. *J. Fluid Mech.* **51**, Pt 1, 187–205, 1972.
9. LIN S. P., Bubble coalescence in fluidized beds. *AIChE Journal* **16**, 1, 130–133, 1970.
10. CLIFT R. and GRACE J. R., Bubble interaction in fluidized beds. *Chem. Eng. Progr. Symp. Ser.* **66**, 105, 14–27, 1970.
11. CLIFT R. and GRACE J. R., Bubble coalescence in fluidized beds. *AIChE Journal* **17**, 1, 252–255, 1971.
12. LAVRENT'YEV M. A. and SHABAT B. V., *Methods of the Theory of Functions of a Complex Variable*. Nauka, Moscow, 1965.
13. BOBKOV N. N. and GUPALO Yu. P., The effect of splitting and coalescence of bubbles on mass transfer in a fluidized bed. *Prikl. Mat. Mekh.* **48**, 5, 782–792, 1984.
14. BOBKOV N. N. and GUPALO Yu. P., A packet mechanism of mixing in a fluidized bed. *Izv. Akad. Nauk SSSR, MZhG* 5, 73–84, 1983.
15. BOBKOV N. N. and GUPALO Yu. P., The unsteady motions of local inhomogeneities in a fluidized bed. *Prikl. Mat. Mekh.* **52**, 3, 431–443, 1988.
16. VAISMAN A. M. and GOL'DSHTIK M. A., A dynamic model of the fluid flow in a porous medium. *Izv. Akad. Nauk SSSR, MZhG* 6, 89–95, 1978.

17. YAMAMOTO K. and YOSHIDA Z., Flow through a porous wall with convective acceleration. *J. Phys. Soc. Jap.* **37**, 3, 774–779, 1974.
18. BOBKOV N. N. and GUPALO Yu. P., Inertial effects at solid phase concentration discontinuities in a disperse medium. *Prikl. Mat. Mekh.* **50**, 5, 758–771, 1986.
19. SOBOLEV S. L., *The Equations of Mathematical Physics*. Nauka, Moscow, 1966.
20. KANTOROVICH L. V. and KRYLOV V. I., *Approximate Methods of Higher Analysis*. Fizmatgiz, Moscow, 1962.
21. DAVIDSON J. F. and HARRISON D., *Fluidised Particles*. Cambridge University Press, Cambridge, 1963.
22. TIKHONOV A. N. and SAMARSKII A. A., *The Equations of Mathematical Physics*. Nauka, Moscow, 1977.
23. LAVRENT'YEV M. A. and SHABAT B. V., *Problems of Hydrodynamics and their Mathematical Models*. Nauka, Moscow, 1977.
24. DAVIES R. M. and TAYLOR G., The mechanics of large bubbles rising through extended liquids and through liquids in tubes. *Proc. Roy. Soc.* **A200**, 1062, 375–390, 1950.
25. KOPPENFELS W. and STAHLMANN F., *The Practice of Conformal Mappings*. IIL, Moscow, 1963.
26. DAVIDSON J. F., HARRISON D. and GUEDES DE CARVALHO J. R. F., On the liquidlike behavior of fluidized beds. *Annual Review of Fluid Mechanics* **9**, 55–86, 1977.
27. GOLUZIN G. M., *Geometric Theory of Functions of the Complex Variable*. Nauka, Moscow, 1966.
28. SEDOV L. I., *The Mechanics of a Continuous Medium*, Vol. 2. Nauka, Moscow, 1984.

Translated by E.L.S.